Debt Capacity and the Anatomy of Sovereign Crises

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Sovereign defaults are rarely complete

- Governments in distress keep paying some obligations and suspend others.
- Missed payments accumulate as arrears that add to the debt.
- Countries often leave a default episode owing more than when it began.

Arellano, Mateos-Planas, and Ríos-Rull (2023): a theory of partial default.

- The sovereign chooses what share of its debt service to withhold.
- It never loses market access. The price reflects the expected default intensity.
- Data: in partial default a third of the time, withholding $\sim$40% of debt service.

What does partial default teach us about sovereign crises?

Four lessons.

1. Debt capacity. Set by the cost of defaulting, not the value of market access.
2. Entry. Defaults begin with a jump, when shocks overwhelm a precautionary buffer.
3. The episode. A financing regime: borrowing continues until output recovers.
4. Pessimism. Lenders' expectations alone can produce a slow crisis, without a run.

Three versions of the model: deterministic, a Markov chain, and a calibrated diffusion. Continuous time makes it tractable: no stopping problem, a static default decision.

The Model

The sovereign

- Preferences $\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t)\, dt$. Log endowment z_t, a Markov process.
- Long-term debt B_t: coupon λ, maturity rate δ, debt service $(\delta + \lambda)B_t$.
- Default intensity $d_t \in [0,1]$: share withheld. A fraction κ becomes arrears.

$$c_t = \phi(d_t, z_t)\, e^{z_t} - (1 - d_t)(\delta + \lambda)B_t + q_t\, \ell_t, \qquad \dot{B}_t = \ell_t - \delta B_t + \kappa(\delta + \lambda)\, d_t B_t$$

- Output cost of default: variable and fixed,

$$\phi(d, z) = (1 - \gamma_0 d^{\gamma_1})\, F(z) \text{ for } d > 0, \qquad \phi(0, z) = 1.$$

- Competitive risk-neutral lenders price the expected default path, $q(B, z)$.

Arrears are a source of funds at a fixed price

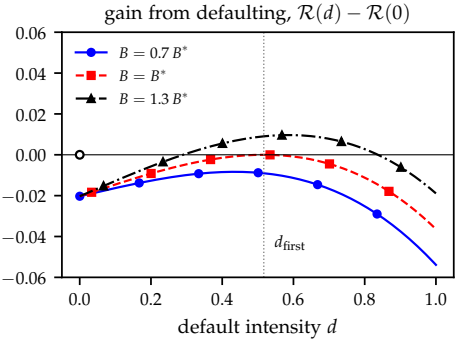
Let n_t be the total addition to debt, from issuance and arrears. Then

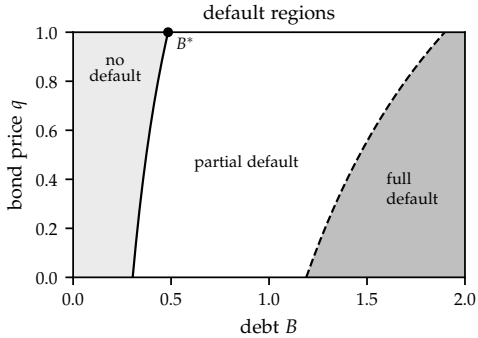
$$c_t = \phi(d_t, z_t)e^{z_t} - (\delta + \lambda)B_t + q_t\, n_t + (1 - \kappa q_t)(\delta + \lambda)B_t\, d_t.$$

- Withholding a unit of debt service raises consumption by $1 - \kappa q_t$, at an output cost.
- Its price is fixed by the recovery on arrears, a haircut that existing creditors bear.
- Its terms do not deteriorate with the sovereign's creditworthiness.
- So arrears are attractive precisely when market prices are low.

Lenders price expected withholding. No transfer: the output cost is a deadweight loss.

The default decision is static





- Given the price, share withheld maximizes $\mathcal{R}(d; B, z, q)$, independent of V.
- The regions of no, partial, and full default are explicit in the state and the price.

Debt Capacity

Lesson 1: the cost of defaulting sets the debt limit

Theorem (debt capacity in closed form)
In the deterministic model the impatient sovereign borrows at the risk-free price up to

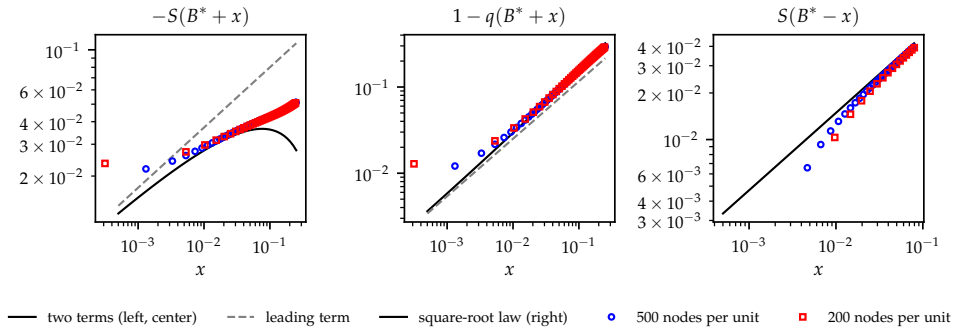
$$B^* = B_{\text{pd}}(0, q^{\text{rf}}),$$

the largest debt it is willing to service in full at the risk-free price, and stops there.

- Closed form in the default costs, the recovery on arrears, and debt maturity.
- Independent of patience, risk aversion, and the borrowing limit.
- Higher with larger default costs, a higher recovery, and longer maturity.

Preferences decide whether to go to the limit. The cost of defaulting decides where it is.

Around capacity: a cube-root law



- Above capacity: repayment speed $\propto x^{1/3}$, price gap $q^{\text{rf}} - q \propto x^{2/3}$ (vertical tangent).
- From below, debt reaches B^* in finite time, along a square-root law.
- An overhang is priced at once and worked off slowly.

How Defaults Begin

Lesson 2: a sovereign never defaults a little

- The intensity is a continuous choice, but the fixed cost makes entry lumpy.
- The first default is a jump of a size set by the costs alone,

$$d_{\text{first}}(z) = \left[\frac{1 - F(z)}{(\gamma_1 - 1)\gamma_0 F(z)}\right]^{1/\gamma_1},$$

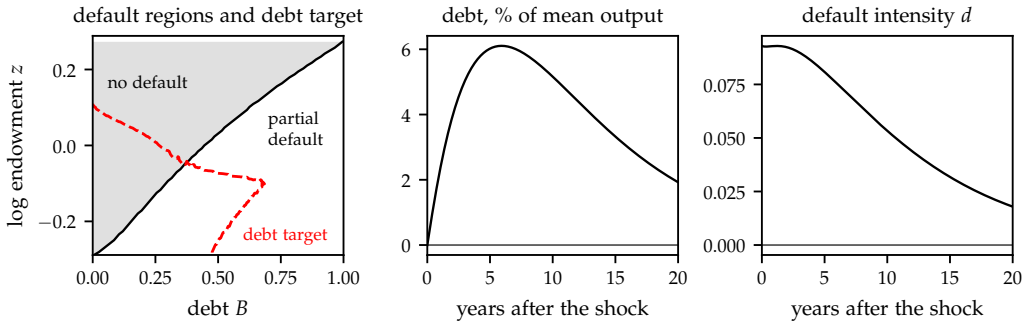
  about half of the debt service at the calibrated parameters ($d_{\text{first}} \approx 0.52$).
- Fixed cost makes full repayment possible. Steep variable cost makes the jump large.

What triggers it? Uncertainty, through precaution.

- The sovereign holds a buffer below its threshold, set by an Euler equation.
- Default occurs when a shock moves the threshold below the debt, past the buffer.

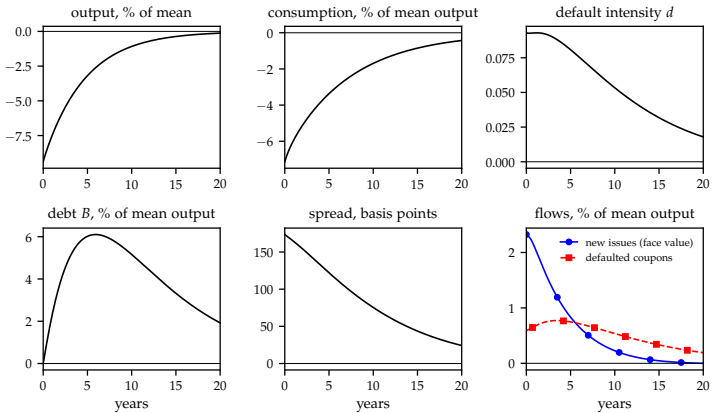
The Anatomy of a Crisis

Lesson 3: the episode is a financing regime



- In bad times the fixed cost is paid regardless and no longer restrains borrowing.
- Debt target set by dilution of the sovereign's own price, higher than in good times.

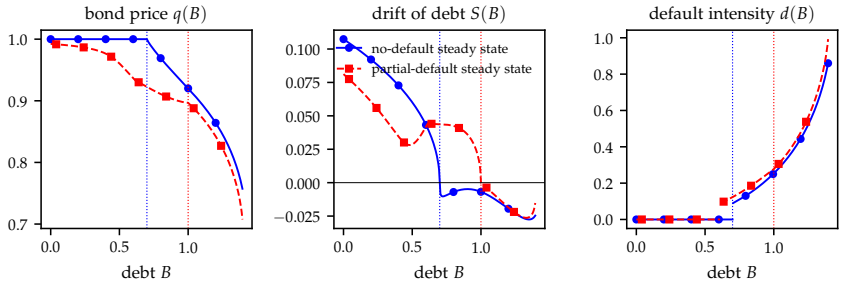
After a bad shock



- Shock smoothed by issuing and withholding.
- To the peak: issues add 8.4, arrears 1.6 (% of output).
- Episode ends when output recovers, not when debt is repaid.

Self-Fulfilling Partial Default

Lesson 4: pessimism alone can produce a slow crisis



- Lenders expect a permanent partial default above capacity.
- Lower price: cheaper withholding, and borrowing past the old limit.
- It drifts into permanent partial default over years. No run, no loss of market access.

When is pessimism self-fulfilling?

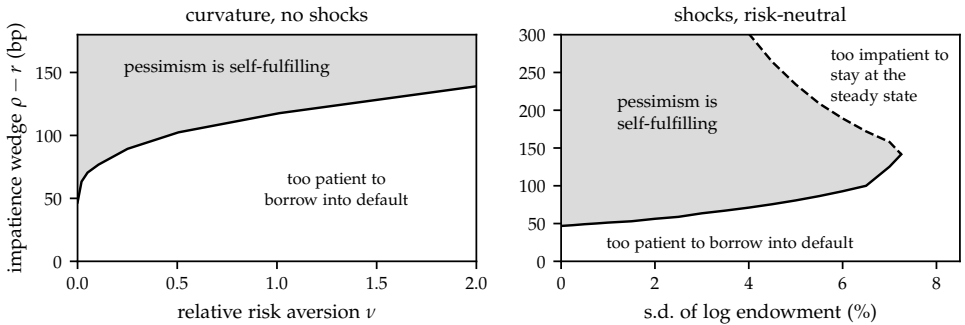
Proposition (risk-neutral sovereign)
A self-fulfilling partial default can be reached from zero debt only if the impatience wedge covers the output cost of the cheapest default per unit of funds raised, $\rho - r \geq \Theta^*$. When the default cost is not too convex,

$$\Theta^* = s_{\text{first}},$$

the spread implied by withholding the first default intensity forever. Equilibria then exist for a range of wedges above it.

- The candidates form a continuum. Lenders' expectations alone select among them.
- In the calibrated model the wedge is 225 bp against a threshold of 1,286 bp.

Curvature and uncertainty raise the bar



- Risk aversion of two roughly triples the impatience needed (139 vs. 47 bp).
- Shocks: window narrows from both sides, closes at endowment s.d. $\approx 7\%$.

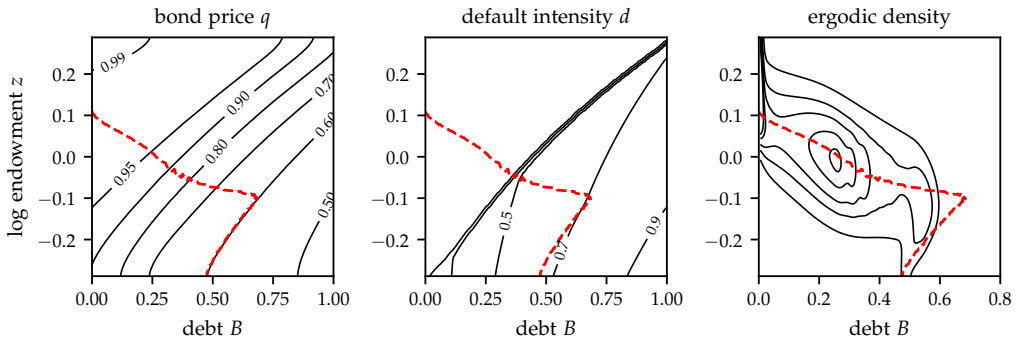
Quantitative Model

Calibration

Targets, free parameters, and weights of Arellano, Mateos-Planas, and Ríos-Rull. Loss 0.077 vs. 0.228.

Moment	Weight	Data	AMR	Model
Targeted				
Partial default: frequency	1	0.36	0.37	0.38
Partial default: mean $\mid d > 0$	1	0.38	0.39	0.41
Partial default: s.d. $\mid d > 0$	1	0.22	0.19	0.22
Debt to output: mean	1	0.32	0.32	0.27
Debt to output: s.d.	1	0.18	0.25	0.18
Debt service to output: mean	1	0.036	0.035	0.037
Debt service to output: s.d.	1	0.021	0.022	0.020
Debt due to output: mean	3	0.049	0.050	0.052
Spread: s.d.	1.25	0.041	0.037	0.035
Output: persistence	1	0.89	0.88	0.87
Output: s.d.	1	0.10	0.12	0.11
Loss			0.228	0.077
Untargeted				
Defaulted coupons to output: mean $\mid d > 0$		0.052	0.040	0.040
Defaulted coupons to output: s.d. $\mid d > 0$		0.064	0.036	0.027
Spread: mean		0.053	0.016	0.029
Spread: correlation with output		-0.17	-0.38	-0.78
Spread: correlation with debt		0.24	0.56	0.87
Consumption s.d. relative to output s.d.		1.00	0.91	0.93

The default regions and the ergodic set



- No-default region: a wedge in (B, z). Mass concentrates along the zero-drift locus.

Computation

- One HJB and one linear pricing equation describe the equilibrium everywhere.
- Implicit upwind scheme, monotone treatment of the fixed-cost non-concavity.
- No taste shocks or other smoothing: the computed decision is the model's own.
- Moments and impulse responses from the generator, not from simulation.
- A parameter evaluation takes minutes.
- An underwriting standard on new issues makes the borrowing limit irrelevant.

Conclusion

Takeaways

- Arrears raise funds at a price fixed by the recovery, a haircut creditors bear.
- Debt capacity is a closed form in the cost of defaulting, not in preferences.
- Crises begin with a jump, when shocks overwhelm a precautionary buffer.
- The episode is a financing regime: borrowing continues until output recovers.
- Pessimism can cause a slow crisis without a run. Curvature, shocks raise the bar.